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Fair sharing in learning rational numbers in a 5th grade class

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Keywords

Learning, teaching, rational numbers, fair sharing

Abstract

Rational numbers are considered a difficult set to teach and learn, and these difficulties are identified in the literature as related to the nature of the set itself, as well as to its teaching. Thus, one of the difficulties relates to the fact that the more traditional teaching does not take into account the different meanings of fractions and the diversity of units. Thus, it is important to carry out studies that promote opportunities for students to work on those ideas. This study aimed to understand the implications of fair sharing tasks in the learning of rational numbers in an exploratory teaching context, in a 5th grade class. To achieve this goal a didactic sequence composed by 7 tasks was implemented, but in this article only the results of 4 tasks shall be analysed. To carry out the study the interpretative paradigm was used, with essentially qualitative approach, case study design, and the sequence of tasks implemented. Data were collected through participant observation and student productions. From the results emanates, in an initial phase, the use of iconic representation and informal language, that is, the use of informal strategies. However, these culminated with the symbolic representation, namely, the fractional representation, so the students seem to have managed to connect their intuitive strategies with the symbolic representation, in a meaningful way. Thus, the context of fair sharing seems to have promoted an informal approach to situations and therefore the connection between informal and formal ideas in a meaningful learning process.

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Introduction

The specialty literature (e.g. Mamede, Pinto & Monteiro, 2020) mentions that rational numbers are a difficult content to teach and learn. The difficulties arise from the expansion of the sense of number, which causes some conceptual conflicts in children, namely those related to the density of the set, and also from the way this content is worked, usually through algorithms and rules devoid of meaning and/or context. In order to overcome these difficulties, several authors (e.g. Mamede, Ribeiro and Pinto (2020)), Monteiro and Pinto (2007), Pinto and Mamede (2019)) emphasise the importance of exploring the different meanings of fractions, when introducing rational numbers, through fair share problems with a real context close to the student. In this sense, this study was conducted with the aim of understanding the strategies and difficulties students have in solving fair share tasks in an exploratory teaching context. According to Canavarro (2011), in this approach, students learn from the exploration of tasks, which bring out the need or advantage of mathematical ideas, systematised in collective discussions. Thus, students develop their learning in meaningful contexts, which promote their involvement and, therefore, an active performance through rich mathematical discussions.

This article begins by presenting a literature review on the teaching and learning of rational numbers in contexts that promote significant learning. It is followed by the methodology adopted in the research, a discussion of the main results, which emanated from the lesson developed with a 5th grade class and, finally, some final remarks.

Literature review

The teaching and learning of rational numbers

The key lessons for the 5th grade of the 2nd cycle of Basic Education (ME, 2018) in Portugal, recommend a deepening of the study of non-negative rational numbers, in the decimal representation and in the fraction form, and the introduction of the representations in percentage and mixed numeral. Therefore, it is important that in the early years of schooling the rational numbers are developed with understanding, since "they constitute one of the subjects of basic education that will have more repercussion in the understanding of key subjects of school mathematics" (Monteiro & Pinto, 2007, p.15). Vamvakoussi (2015) also values the understanding of rational numbers at these ages, given that "the notion of the unit and of the arithmetical operations need to be reconceptualised; and there are several conceptually distinct meanings attached to rational numbers that, again, need to be understood and coordinated." (p. 50). Thus, in order to promote a development with understanding of rational numbers, it is necessary to overcome conceptual conflicts that arise in the study of these numbers.

According to Monteiro and Pinto (2007), one of the struggles in teaching and learning rational numbers is inherent to the set itself, namely because it is a dense set, i.e., between any two numbers there is an infinity of numbers, which can be represented in different ways (fraction, decimal numeral), which does not happen with whole numbers and hinders the understanding of rational numbers. It should be noted that this is the first set of numbers that students encounter that is not based on the counting process (Cid, Godino & Batanero, 2004).

Another difficulty is related to the teaching of rational numbers, which according to Monteiro and Pinto (2007), is limited to the transmission of rules and algorithms devoid of context or meaning. The authors also point out the fact that more traditional teaching does not take into account the different meanings of fractions and the diversity of units, but "where fractions and decimal numbers are presented through a unit divided into a certain number of parts (10, in the case of decimals), the respective representation is presented and from there on paper and pencil calculation follows, without encouraging mental calculation and estimation either (p.17). Consequently, students are left without the quantitative notion of rational number, that is, with the notion that these are also numbers, but that they can be represented in different ways as decimal numbers, ratios, divisions, points of a number line, measures and parts of a whole, as found by Post, Behr and Lesh (1986). Thus, it is suggested to create problematic situations, which have the students' everyday lives as context, and to give them the opportunity to solve them based on the knowledge they have, making connections in a natural and meaningful way (Monteiro & Pinto, 2005).

Research data (e.g. Mamede, Pinto & Monteiro, 2020) suggests that students develop an intuitive understanding of rational numbers from experiences involving division. Thus, an initial approach to this topic, based on students' everyday situations, in a fair sharing context, allows for the constructive development of mathematical concepts (Monteiro & Pinto, 2007). Thus, students understand the meaning of "numerator", "denominator" and the relationship between these, making it possible to understand the concept of "fraction"(Pinto & Mamede, 2019).

Therefore, according to Pinto and Mamede (2019) from fair sharing tasks in exploratory teaching context it is possible to create meaningful learning paths for students that involves (i) the exploration of meaningful tasks close to their everyday life in a fair sharing context; (ii) the creation of their own informal strategies, i.e., an active participation in their learning; and (iii) the discussion of ideas in a large group, promoting discussion and mathematical argumentation, also based on error, in order to construct and reconstruct concepts, thus consolidating their learning.

Methodology

To understand the students' strategies and difficulties in solving fair sharing tasks in an exploratory teaching context, we adopted the interpretative paradigm, with a qualitative approach, in a case study design, since it "provides the opportunity for an aspect of a problem to be studied in depth within a limited period of time" (Ventura, 2007, p. 385).

Two master's degree students and the head teacher of a 5th grade class, where both students were carrying out their Pedagogical Practice, participated in the study. The class where the research was carried out was comprised of 16 students, two of whom with Special Health Needs. Data was collected using audio and video recording of the lesson and also the logbook, where observation records were made during the lesson and the students' productions.

Two lessons were conducted in order to initiate the content of rational numbers, through a didactic sequence composed of 7 tasks. Of these, 4 were developed in the first class and 3 in the second research class. In these lessons, the exploratory teaching methodology was used, where each task went through three phases (phase 1 of "launching", phase 2 of "exploration" and phase 3 of "discussion" (Canavarro, 2011)). For the exploration of the tasks, the students were arranged in groups of 4 elements, whose composition was previously thought by the participants of the study, having as a composition criterion, the existence of students with and without difficulties in Mathematics.

This article presents and discusses the productions of the four working groups, trying to understand the strategies used in the exploration phase of the first three tasks of the sequence, as well as the difficulties experienced.

Presentation and discussion of results

Task 1

The first task had as context a school trip and required, in the first point, the fair sharing of 3 sandwiches by 5 colleagues, with the purpose of identifying the amount of sandwiches that each colleague ate and, in the second point, that students identified if each colleague ate more or less than one sandwich. With this task it was intended that the students (i) solve fair sharing problems; (ii) use the language of fractions; (iii) use the different representations of rational numbers (fraction, decimal numeral, percentage); and (iv) compare fractions with the unit.

After launching the task, the exploration phase in small groups followed. At this stage all groups started by using the iconic representation to model the situation. However, three of the four groups used the strategy of dividing the three sandwiches into two equal parts and dividing half of the third sandwich into five equal parts. Of the three groups mentioned above, only two (group 3 and 4) also used fractional representation, and only group 3 was able to represent the fraction of sandwiches that each colleague ate, i.e., $\frac{6}{10}$ (Figure 1).

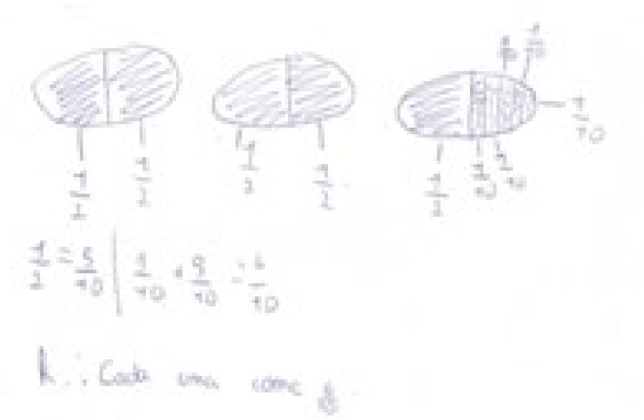


Figure 1. Production of group 3

The use of the iconic representation seems to have led the students of this group to understand that $\frac{1}{5}$ of half of a sandwich corresponds to $\frac{1}{10}$ of the whole sandwich. Therefore, in each half there are $\frac{5}{10}$, so that each colleague had $\frac{5}{10}+\frac{1}{10}=\frac{6}{10}$. Thus, the iconic representation promoted by the context of the task seems to have promoted the need to divide the whole into the same parts and thus, the significant understanding of the equivalence of fractions. Thus, the addition of fractions with the same denominator emanated intuitively and full of meaning. These results are in line with what Pinto and Mamede, (2019) refer when they highlight the importance of fair sharing tasks in exploratory teaching contexts.

Group 4, who also divided half of the third sandwich into 5 equal parts and identified each of these parts as being 0.1 (one tenth), do not seem to have realised that $\frac{1}{2}$ represents the same quantity as 0.5 (five tenths), i.e. they were unable to make the connection between the different representations. This led them to resort to the division algorithm to identify which part of the sandwich each classmate ate (Figure 2).

Figure 2. Production of group 4

Therefore, these students in group 4 seem to have been exposed to a teaching approach limited to the transmission of rules and algorithms devoid of context or meaning, results which are in line with those reported by Monteiro and Pinto (2007).

Group 2, despite having reached the same results as group 3, did not use fractional representation, but decimal representation to represent the amount of sandwiches each colleague ate. Thus, they do not seem familiar with the language of fractions.

Group 1, during the exploration phase of the task, although they had thought of using the same strategy as the other groups, that is, dividing each of the three sandwiches into two equal parts, concluded that with this division "there will be one half left!"

Teacher - What are you thinking, tell me? Why are you thinking there will be any left over?

Student D - Because dividing in half will leave over ($\frac{1}{2}$ over).

Teacher - What will be left over?

Student D - One.

Teacher - A whole sandwich?

Student D - No, half.

Teacher - Half. And then that half, what do you do about it?

(Student D wonders)

Teacher - You were saying divide in half, right D?

Student D - So, divide in half and then whatever is left over we divide by 5.

At this stage the teacher leaves the group to work on their own, later when she returns the students had divided each sandwich into ten equal parts but claimed that each classmate was going to eat two little bits (half, $\frac{1}{2}$, plus $\frac{1}{10}$), without being able to quantify "the two slices" as they claimed, i.e. how much each classmate was going to eat. The teacher intervenes:

Teacher - (...) But if I now want to know how much [each classmate ate] *(...) What is the number that represents the quantity? Is there a number? You said you are going to eat 2 pieces (half* *plus $\frac{1}{10}$), but I don't know the size, if it is this size ($\frac{1}{2}$), if it is that size ($\frac{1}{10}$) and I need to know. How do I do this?*

Student D - Each of you is going to eat 6.

Teacher - 6 pieces, yes, and then? 6 pieces of how many?

Student D - 6 pieces of 30.

(...)

Teacher – Did you divided the sandwich in 30?

Student D - No, in 10.

Teacher - In 10 and you said they will eat 6.

Student D - On each loaf of bread.

Teacher - On each loaf of bread, so, but you're telling me it's 30.

Student M - Yes, because if we add up the 3 loaves of bread it will be 30.

Teacher - Yes, but we have to think about whether our unit is one loaf of bread or whether it's the 3 loaves of bread. What am I dividing?

Student D - 1 loaf of bread.

Teacher - 1 loaf of bread. The 3 is for the 5. I want to know how much one eats.

Student D - 6.

(...)

Teacher - You eat 6 pieces of how many?

Student D: Out of 10.

Teacher - From 10, ok.

Student M - That's a fraction!

Teacher - That's right, now what?

Student M and Student D - 6 over 10.

(...)

Teacher - So how much do they eat?

Student M - Ah, six tenths! (Figure 3)

At this point the teacher asks the students to record their answers on the worksheet and student M, dictates the answer "Each girl eats 2 slices of each loaf of bread ($\frac{6}{10}$)" (Figure 3).

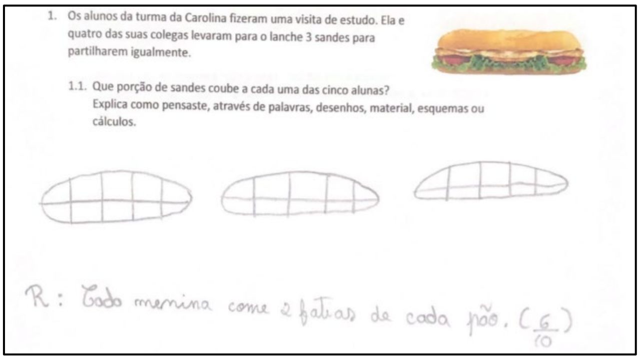


Figure 3. Production of group 1

Thus, based on the question, the teacher helped the students to overcome the misunderstanding about the unit of reference they had to consider, one sandwich and not the three sandwiches, and guided them to answer what was intended, the amount of sandwiches each colleague ate.

Regarding paragraph 1.2 of task 1, all groups concluded that each colleague ate less than one sandwich, basing their reasoning on the iconic representations produced for the resolution of paragraph 1.1, which make the answer to this question very clear.

Group 3, in addition to the informal justification, advances a more formal justification, justifying that $\frac{6}{10}$is less than $\frac{10}{10}$, since this quantity represents the whole (Figure 4).

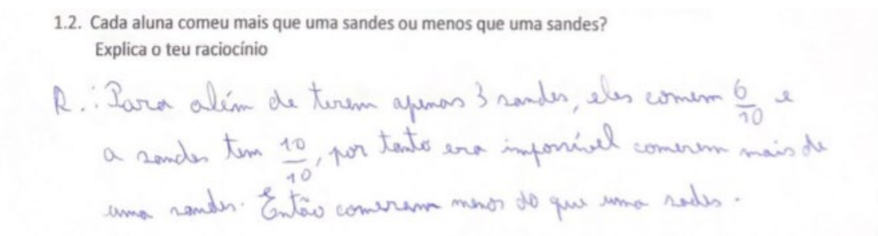


Figure 4. Production of group 3

Thus, all groups seemed to have found it easier to compare fractions with the unit, which was not unrelated to the real context of the task and the consequent iconic representation, which seems to have supported the students' reasoning. These results are in line with those reported by Pinto and Mamede (2019).

In the phase of presentation and discussion of task 1 in the class group, the third phase of exploratory teaching, the different resolution strategies of the groups were confronted, as well as the different symbolic representations of rational numbers (fraction, decimal numeral and percentage). Consequently, the students solved problems of fair sharing, used the language of fractions, the different representations of rational numbers and compared fractions with the unit, thus achieving the objectives that were intended with the resolution of task 1.

It should be noted that most students in the class were not familiar with this type of task, nor with the methodology adopted, the exploratory teaching, where they are responsible for their learning.

Task 2

The second task with the same context as the first one, a school trip, but this time, in the first point, the students were asked to fair share 6 sandwiches among 10 classmates with the purpose of identifying the amount of sandwiches that each classmate ate and, in the second point, the students were asked to identify if each classmate ate more or less than one sandwich. With this task, as in the previous one, it was intended that students (i) solve fair sharing problems; (ii) use the language of fractions; (iii) use the different representations of rational numbers (fraction, decimal numeral, percentage); and (iv) compare fractions with the unit.

Also to solve the first paragraph of task 2, all groups started by using the iconic representation and the same strategy, that is, they divided five sandwiches in two equal parts and the sixth sandwich in ten equal parts. All groups also used the fractional representation, and most groups (2, 3 and 4) used the formal algorithm of addition of fractions with different denominators (e. g. Figure 5). Thus, the students seem to have perceived in a significant way the equivalence of fractions, through the need of having the whole divided in the same parts, to add these parts.

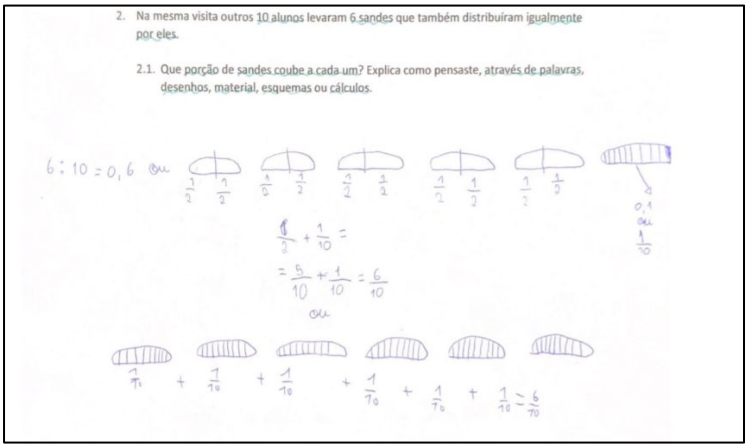


Figure 5. Production of group 4

Group 4 also used the division algorithm, as they had already done in solving task 1 (Figure 5). However, this time they seem to have managed to connect the algorithm to their informal strategies, assigning meaning to it. These results reinforce research data (e.g. Mamede, Pinto & Monteiro, 2020), which suggests that students develop an intuitive understanding of rational numbers from experiences involving division.

Group 1 also presented the connection between fractional representation and percentages (Figure 6).

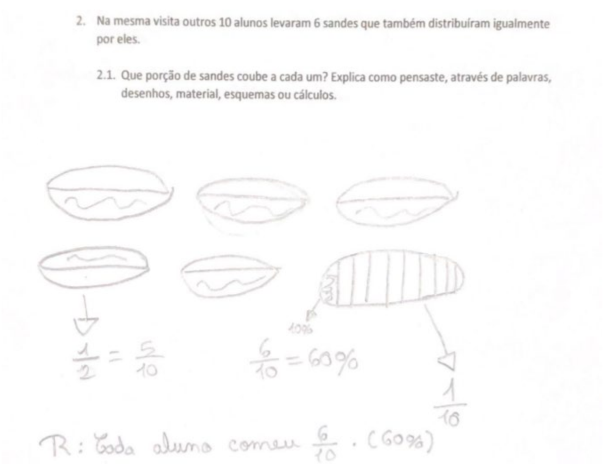


Figure 6. Production of group 1

It should be noted that the representation in percentage appeared in the discussion stage of task 1, so these students seem to have appropriated that representation in a significant way.

In the resolution of paragraph 2.2 of task 2, all groups concluded that each colleague ate less than one sandwich, once again supporting their reasoning on the iconic representations produced for the resolution of paragraph 2.1, which make the answer to this question very clear.

This time it was group 1, which in addition to an informal justification, advanced a more formal one, justifying symbolically that $\frac{6}{10}$is less than $\frac{10}{10}$ (Figure 7).

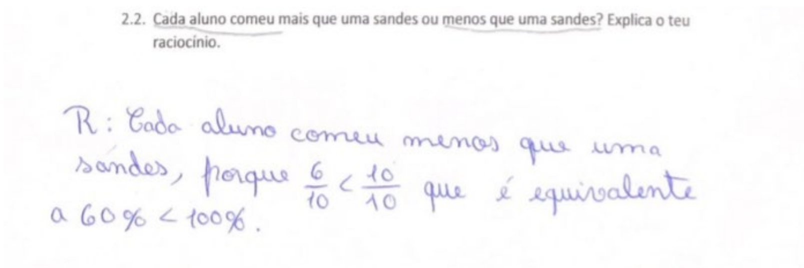


Figure 7. Production of group 1

Group 1, also presented different representations of rational numbers, showing some ease in connecting the fractional representation with the percentage representation.

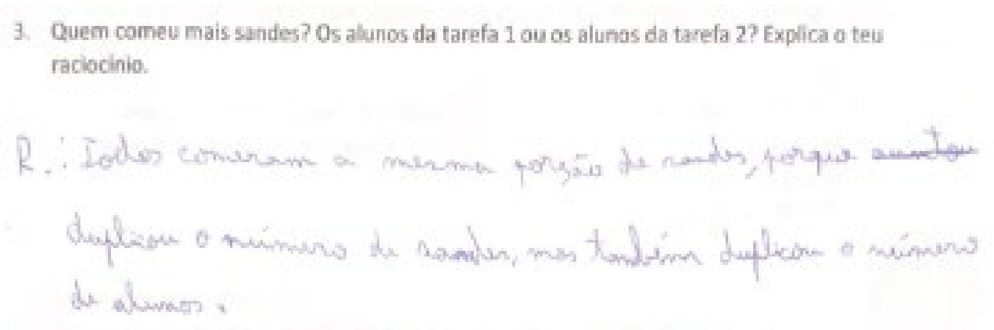
Once again, students seem to have found it easy to compare fractions with the unit, and the use of iconic representation seems to have supported their reasoning, results that are in line with those reported by Pinto and Mamede (2019).

In the presentation and discussion stage of task 2 in the class group, the third phase of the exploratory teaching, the different resolution strategies of the groups were compared, as well as the different symbolic representations of rational numbers (fraction, decimal numeral and percentage). In this way, the students solved problems of fair sharing, used the language of fractions, the different representations of rational numbers and compared fractions with the unit, thus achieving the objectives that were intended with the resolution of task 2.

Task 3

The third task was intended for students to compare the amount of sandwiches that each classmate ate in task 1, with the amount of sandwiches that each classmate ate in task 2. This task had as objectives the comparison of fractions and the recognition of equivalent fractions.

All groups concluded that each classmate in task 1 ate the same amount of sandwiches as each classmate in task 2, since from one task to the other, they doubled the number of sandwiches, but also doubled the number of classmates (e.g. Figure 8).


3. Quem comeu mais sandes? Os alunos da tarefa 1 ou os alunos da tarefa 2? Explica o teu raciocínio.

R.: Todos comeram a mesma porção de sandes, porque aumentou duplicou o número de sandes, mas também duplicou o número de alunos.

Figure 8. Production of group 3

Thus, students arrived at the equivalence of fractions intuitively, based on their own productions to interpret the real context, namely the iconic representations, which supported their reasoning. These results are in line with those presented by Monteiro and Pinto (2007).

The discussion of task 3 in the class group, third phase of the exploratory teaching, allowed the confrontation of different resolution strategies of the groups, as well as the different symbolic representations of rational numbers and the connection between language and informal procedures and language and more formal procedures, namely symbolic ones. In this way, students compared fractions and recognised the equivalence of fractions.

Final remarks

While solving the fair sharing tasks, students first used iconic representations to model the situations described. These iconic representations seem to have supported the reasoning that led them to symbolic representations and, therefore, to more formal representations, namely the different representations of rational numbers (fraction, decimal numeral and percentage). This allowed the constructive development of mathematical concepts. The main difficulty seems to have been, at an early stage, the lack of familiarity with this kind of tasks.

These results seem to confirm the importance of fair sharing tasks, as highlighted by several studies in the area (e.g. Mamede, 2020; Pinto & Monteiro, 2020), because it is a context close to the students and meaningful. This allows them to model situations and thus connect their informal knowledge with more formal knowledge, in a progressive and participatory way in their learning process. Thus, students develop their learning in meaningful contexts that promote their involvement and therefore active performance through rich mathematical discussions.

Given that the set of rational numbers is a difficult subject to teach and learn, it is suggested the continuation of research in the teaching and learning of these numbers, particularly using teaching methodologies such as exploratory, where students are involved in an active/constructive way in their learning process - even more when the use of this work methodology, so promising, remains a limitation in investigations like this one, due to the fact that the students and the teacher are not familiar with it.

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